Challenging Mathematical Gifted Primary Students

Carmel M. Diezmann
Queensland University of Technology

Mathematically gifted students and students with learning difficulties in mathematics differ substantively. However, they are both "at risk" of underachieving in mathematics without an adequately challenging learning environment. This paper explores the characteristics of the mathematically gifted and the implications of these characteristics for curriculum and instruction. It also presents a range of strategies that have been successfully used in regular classrooms to provide the cognitive challenge needed by mathematically gifted students. These strategies include (1) problematising tasks by inserting obstacles to the solution, limiting problem information or requiring students to use particular representations or solution strategies; (2) implementing mathematical investigations to encourage students to apply and create mathematical knowledge in posing and solving novel problems; (3) extending manipulative use to capitalise on visual-spatial or kinaesthetic representations and to support higher-level thinking; and (4) modifying educational games to provide rich mathematical and social learning opportunities. The inclusion of these strategies should contribute to the provision of high quality learning opportunities for the mathematically gifted.

The importance of educating mathematically gifted students has been acknowledged widely in Australia. For example, Howard (2001) stated "we are seeking today to nurture a new generation of young scientific minds capable of achieving great things for their country". At the societal level, the achievement of this goal is urgent because Australia is currently producing inadequate numbers of students who have high level capability in the mathematical sciences (Thomas, 2000). At the individual level, this goal is also urgent because current educational provisions for gifted children in Australia can result in negative outcomes (Collins, 2001):

> [Gifted] children have special needs in the educational system; for many their needs are not met; and many suffer underachievement, boredom, frustration, and psychological distress as a result …. The common belief that the gifted do not need special help because they will succeed anyway is contradicted by many studies of underachievement and demotivation among gifted children. (p. xiii)

Thus, there is a need for education to support the development of mathematical aptitude into mathematical achievement.

According to Gagné (1985, 1993) informal/formal learning and practicing is one of the four components that contributes to the development of aptitude into achievement. The other components are intrapersonal catalysts (i.e., physical, motivation, volition, self-management, and personality), environmental catalysts (i.e, millieu, persons, provisions, and events), and chance. This paper focuses on ways to support mathematically gifted primary students in formal learning situations. Firstly, a brief overview of the characteristics and education of mathematically gifted students is presented. This is followed by a discussion of balancing learning and practising in the curriculum, the role of challenging tasks in learning, and instructional strategies that support mathematically gifted students in the regular classroom. This paper concludes with comments about the advantages of the instructional strategies discussed and other issues related to mathematically gifted primary students.

The Characteristics and Education of Mathematically Gifted Students

Exceptional reasoning ability is the distinguishing characteristics of mathematically gifted students (House, 1987; Johnson, 1983). Additionally, they may display their aptitude for mathematics in a range of other characteristics (See Table 1). Because mathematically gifted students may draw on either logico-mathematical intelligence and/or spatial intelligence, there are three distinct types of mathematically gifted students, namely analytic thinkers, geometric thinkers, and harmonic thinkers

(Kruteskii, 1976). Analytic thinkers have high logico-mathematical ability and weaker spatial ability. Geometric thinkers have high spatial ability and weaker logico-mathematical ability. Harmonic thinkers have high logico-mathematical ability and high spatial ability. Thus, gifted students can vary substantively in their interest and achievement on particular types of mathematical tasks.

Table 1.
Characteristics of Mathematically Gifted Students

- Has exceptional reasoning ability and memory.
- Has a tendency to choose to do mathematics when presented with a choice of activities.
- Masters typical content more quickly and at an earlier age than his or her classmates.
- Often skips steps in problem solving and may solve problems in unexpected ways.
- Is more willingly and capable of doing problems abstractly: often prefers not to use concrete aids.
- Enjoys and is successful looking for patterns and relationships and attempts to explain them.
- Concentrates for long periods of time on a problem that he or she finds interesting.
- Is more likely to see relationships between a new problem and problems previously solved; enjoys posing original problems.
- Is capable of more independent, self-directed activities.
- Enjoys the challenge of mathematical puzzles and games.

(House, 1987, pp. 51-52)

Various educational strategies are advocated for mathematically gifted primary students (e.g., acceleration, enrichment programs, curriculum compacting, competitions). Although these strategies can be successful, they may also be ad hoc throughout a student's schooling, inaccessible to some students, or fail to adequately cater for particular types of gifted students (e.g., underachievers, girls, minority groups). Therefore, of particular importance is how teachers can provide rich mathematical learning opportunities within the regular classroom. Rich learning opportunities stem from students' engagement in challenging tasks (Diezmann & Watters, 2000, 2002b; Lupkowski & Assouline, 1992; House, 1987; Sheffield, 1999). However, many tasks that gifted students undertake in the regular classroom provide few opportunities for learning.

Balancing Learning and Practising in the Curriculum

Classroom tasks can be broadly categorised into those that have the potential for learning and those that provide opportunities for students to practice applying their previously learnt knowledge or skills. Whether the outcome of a task is "learning" or "practising" is dependent on the ability of the student who is engaged in that task. For example, a task that is designed to encourage students to explore a set of number facts has "learning value" for students who are unfamiliar with those number facts but only "practice value" for students who know the particular number facts. Practice with number facts will presumably improve speed and accuracy. Figure 1 illustrates the balance of learning and practice that is appropriate for non-gifted students (Group A) and gifted students (Group B). No numerical relationships should be inferred from this figure which has been produced for illustrative purposes only. The sloped line indicates learning and the horizontal line indicates practice. The trajectory for the non-gifted students (Group A) was produced to illustrate commencing with students' prior knowledge (Point 1), allowing some time for a variety of learning experiences (Point 5), and further time to practice for mastery of the content (Point 6). The trajectory for the gifted students (Group B) was hypothesised by incorporating what is known about the learning of gifted students (e.g., Davis & Rimm, 2004; House, 1987). At a particular point in time their commencing knowledge of a topic is likely to be greater than their non-gifted peers (Point 1). Thus, it may be some time into a new content for non-gifted students that gifted students have the opportunity to build on their prior knowledge (Point 2). Gifted students learn more rapidly than their non-gifted peers. Hence, they will reach a point where the value of a task moves from having learning value to practice value more quickly than their non-gifted peers (Point 3). Gifted students need less practice than their gifted peers so they will achieve mastery sooner than their non-gifted peers (Point 4). These students may have an extended practice period if they continue working on tasks that they can easily complete (Point 4 to Point 6).

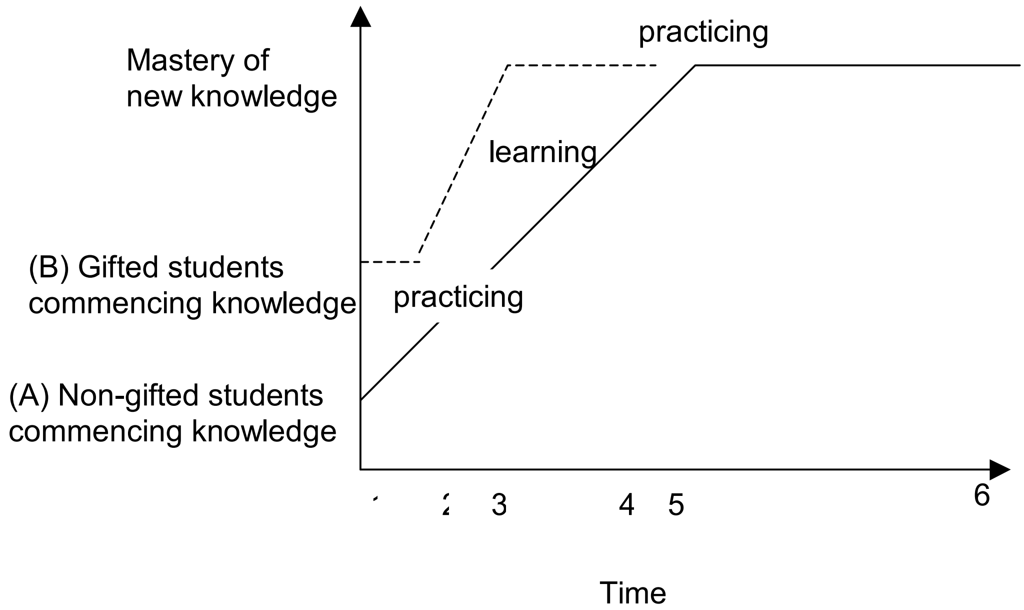


Figure 1. Gifted and non-gifted students mastery of new content knowledge.

Table 2 presents a comparison of the learning and practice time of gifted and non-gifted students based on Figure 1. This table reveals that gifted and non-gifted students in a non-differentiated program experience a substantially different balance between tasks that have learning value and tasks that have practice value. Assuming that the balance between learning and practice tasks is appropriate for non-gifted students, the balance between learning and practice tasks for gifted students is highly problematic for two reasons. Firstly, at the commencement of a new content for non-gifted students in a non-differentiated program, gifted students are engaged in practice tasks rather than learning tasks. This may impact negatively on motivation and interest in the "new content. Secondly, in a non-differentiated program, gifted students spend far more time practising new content than their non-gifted peers (See Figure 1). This is ironic because gifted students need less rather than more practice than their non-gifted peers to achieve mastery. Thus, after gifted students have achieved mastery of new content through practice the value of further practice is negligible and can be counter productive if boredom or frustration results.

Table 2.
Comparing the Learning and Practice of Non-Gifted and Gifted Students

Time	Points 1-2	Points 2-3	Points 3-4	Points 4-5	Points 5-6
Group A - non gifted	**learning**	**learning**	**learning**	**learning**	practice
Group B - gifted	practice	**learning**	practice	practice	practice

Not surprisingly many gifted students in non-differentiated programs complete their work early. Some of these students are given more of the same types of activities, unrelated tasks or employed in tutoring their non-gifted peers. In each situation, the assignment of tasks for gifted students needs to be educationally defensible in terms of either the learning value or the practice value of a task. It is neither educationally defensible to provide busy work to keep a gifted student occupied nor to have them tutor a non-gifted classmate if there is no value in the task for the gifted student. Table 2 highlights the need to shift the focus from practice to learning for gifted students. The following section provides a brief overview on the importance of challenging tasks for learning.

Challenging Tasks

As discussed earlier with the number facts example, it is too simplistic to consider the level of challenging of a task as "set" or "fixed", because the potential for learning from a task varies from individual to individual. Learning occurs when students engage in tasks with high relative cognitive value for him or her (Diezmann & Watters, 2000) and are within the student's Zone of Proximal Development [ZPD] (Vygotsky, 1978). However, the cognitive value of a task can change throughout the life of the task. Henningsen and Stein (1997) propose that tasks exist at four levels and that the transition between these levels moderates the level of thinking and reasoning required on a task (See Table 3). For example, if a teacher provides unnecessary scaffolding on a task the cognitive value can be substantially reduced between being announced by the teacher and implemented by the students. Unnecessary scaffolding occurs when all students receive hints and clues rather than only those who need this support. However, both teachers and students play an important role in the cognitive level of a task (Hiebert et al., 1996): "Tasks are inherently neither problematic nor routine. Whether they become problematic depends on how teachers and students treat them (p. 16)". Thus, there is a need to establish classroom norms in which students develop a commitment to tackling challenging tasks and teachers provide judicious support to students.

Table 3
Conditions Where Task Difficulty Can be Modified

The task as represented in the curriculum or instructional materials.
The task as announced to the students by the teacher.
The task as implemented by the students.
The task as reflected in the products accepted by the teacher.

Instructional Strategies that Support Mathematically Gifted Students in the Regular Classroom

Ideally, mathematically gifted students should have a differentiated program that caters for their unique learning needs and capabilities. However, such a program is inaccessible to many students due to the need for professional training of classroom teachers and time and resource limitations. A more pragmatic approach towards supporting the development of mathematically gifted students at least in the short-term is to identify instructional strategies that capitalise on challenging tasks, accommodate the characteristics of mathematically gifted students, and can be implemented readily in the regular classroom. The focus on challenging tasks shifts the emphasis from tasks that have practice value for gifted students to those that have learning value. In this section, four strategies that meet these criteria follow.

Strategy 1 - Problematising Tasks: One of the most common difficulties with tasks in the regular classroom is that they are too simple for gifted students, and hence, have negligible learning value. Given that gifted students generally master typical content more quickly and at an earlier age than their classmates, a task that is appropriate for non-gifted students is likely to be too easy for gifted students. However, the level of difficulty of a task can be increased through problematisation. This may involve, for example, inserting obstacles to the solution, limiting the problem information provided, increasing the magnitude of the quantities in the problem or requiring students to use particular representations, solution strategies or develop generalisations. For example, after easily calculating the sum of the numbers from 1 to 10, a young gifted student used a generalisation to calculate the numbers from 1 to 100 (Diezmann & Watters, 2000). This task was appropriately challenging for the gifted student but still connected to the regular curriculum focus of summing sequential numbers.

In the following example, a division task with remainders was selected for a class of 11- and 12-year-olds. An inbuilt difficulty in this task was whether or not Justin should be included in the calculations. That is, if the quantity of sweets should be divided by four or five to accommodate the four friends or Justin and his four friends respectively. Lisa and Rachel, two gifted students, completed the regular task rapidly. They assumed that the sweets did not need to be shared with Justin, and hence, divided the number of sweets by the multiples of four that fell between 10 and 20 (See Figure 2, LHS). Thus, this task had low relative cognitive value for these particular students. Even if there had been class agreement that the quantity should have been divided by five rather than four, the relative cognitive value of the task would have been low for these gifted students. Thus, this task had practice value rather than learning value for Lisa and Rachel. The task was problematised by modifying two aspects of the original task. Firstly, the number of sweets was less defined and changed from "between 10 and 20 sweets" in the regular task to "the least number of sweets" in the problematised task. Secondly, the number of people sharing the sweets was modified. In the first task, the number of people could be four or five depending on whether or not students included Justin in their calculations. However, in the problematised version, the phrase "among 4 of his friends" was replaced with "up to 5 friends". This change meant that students needed to search for a number with multiple divisors (See Figure 2, RHS). One further modification was made to the problematised task. This was to eliminate the remainder. Even though the task needed to be more difficult, it still needed to be within students' ZPD. The problematised task involved Lisa and Rachel in considerable discussion about solution strategies. They also used a grid representation to help them keep track of their thinking. Thus, the problematised task had a higher relative cognitive value for these students than the initial task. In problematised tasks, the students may need some teacher support, however scaffolding should be kept to a minimum to maintain the difficulty of the task. Problematised tasks are discussed further elsewhere (Diezmann & Watters, 2002b).

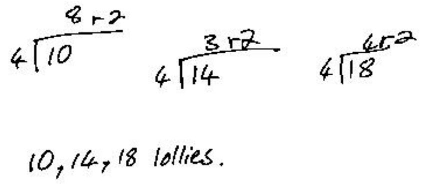
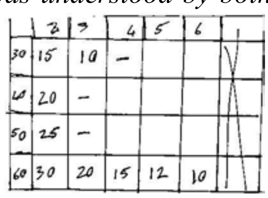
Regular Task: Sweets A	**_Problematised Task: Sweets B_**
Justin has [1]between 10 and 20 sweets. If he shares all of them [2]among 4 of his friends he will have [3]2 left over. How many sweets could he have had?	*What would be the [1]least number of sweets in a bag, if Justin could share his sweets [3]exactly with [2]up to 5 friends?*
 (Lisa & Rachel)	*60 lollies… It said five friends so you didn't really have to do "1" (Going across the diagram) 2, 3, 4, 5, 6. 2 was divisible by 30 (numbers are inverted but meaning was understood by both) it was 15, then it was 10 (moving across). It didn't work (dividing 30 by 4) so we didn't go any farer (sic) than that …. Because it didn't work" (meaning not divisible by 4). (Lisa & Rachel)* 

Figure 2. A regular task and a problematised task.

Strategy 2 - Implementing Mathematical Investigations. Investigations are central to the reforms advocated internationally to improve mathematical learning and develop children's mathematical power (Baroody & Coslick, 1998). Mathematical investigations have learning value for gifted students because they are open-ended and require students to apply and create mathematical knowledge in posing and solving novel problems. Investigations also provide scope for teamwork and productive dialogue through questioning, debate, explanation, and co-construction of meaning. Thus, investigations accommodate the mathematically gifted students' characteristics of long attention spans, interest in exploring relationships and posing original problems, and engaging in self-directed activities.

The following task provided the stimulus for a variety of mathematical investigations for seven- to eight- year old students (See Figure 3, LHS). This task was challenging for all students. Karen and Amy's solution demonstrated exceptional reasoning for this age student. They calculated the amount of money in the pig by using their knowledge of the mass of $3, employing ratio, and comparing the weights of full and empty pigs. This multi-step approach is very sophisticated reasoning for students of this age.

The Problem: The Olympics Trip *I would like to take my friend to Sydney for the Olympics. I have been saving money in my piggybank for a long time. How can I find out how much is in the piggybank without opening it? I cannot open the piggybank without breaking it because the stopper is stuck in the bottom. I only want to break open the piggybank if I am sure I will have enough money. What would you suggest I do?* 	*First we weighed the empty pig. Then we weighed the full pig. Then we weighed the money ($3). We think its $12-$15 because $3 weighs to 1 on the scales we counted in threes until we reached what the full pig weighed (Karen & Amy)* 

Figure 3. A trip to the Olympics.

Strategy 3 - Extending Manipulative Use: Manipulatives are often used in primary schools, particularly the early grades to provide concrete referents for abstract ideas. However, one of the characteristics of mathematically gifted students is their preference not to use manipulatives. This may be because the manipulatives are unnecessary for the task (Marojam, 1992):

> The child who already possesses the ability to add, subtract and multiply in his head can be positively bewildered by being made to perform the operation through his senses—with objects, coloured sticks and blocks. One may as well issue the football team with crutches. (p 41)

Manipulatives can be helpful for gifted students when they capitalises on visual-spatial or kinaesthetic representations to support higher-level thinking. For example, young gifted students construction of a physical number-line using upturned plastic cups to represent the ten brightest stars enhanced their knowledge of large numbers, relative magnitude, and scale (Diezmann & English, 2001). This application of manipulatives goes beyond the more typical use of a prepared number line to represent relatively simple sets of numbers.

The situation where some students may benefit from the use of manipulatives and others may not creates a conundrum for a teacher in a whole class lesson. One teacher addressed this issue effectively by using manipulatives in ways to benefit both non-gifted and gifted students. The context of the use of manipulatives was the story, "The Doorbell Rang" (Hutchins, 1986), which repeatedly features a group of children sharing out cookies. This story commences with a small number of children sharing out a batch of 12 cookies. The doorbell then rings, more children arrive, and the cookies are re-shared amongst all children. The children acted out the story using cookies and plates. The teacher also used a square tablecloth on the carpet to position the actors to sit and share the cookies. She sat the first two children opposite each other with their share of six cookies each on two plates. When the next two children arrived, they sat on the vacant sides of the tablecloth (See Figure 4). The new mathematics for most children was that the size of an individual's share reduces when more individuals share a finite number of objects. The inverse relationship between the number of people and the number of objects they receive is a key understanding in division and helps children judge the reasonableness of their answers in sharing situations. The manipulatives worked very well for this purpose. The teacher also pursued the more advanced ideas of the relationship between the doubling of one quantity and the halving of another with some of the students. The arrangement of the manipulatives and the act of two characters giving the two newcomers half their cookies and the teacher's questioning made this very explicit (For further discussion of this task see Diezmann, Watters, & English, 2002). A number of months later, one capable student used the analogy of cookies to explain that it was possible to share one item between two people. Thus, the manipulatives served to provide capable students with a referent to think beyond their current knowledge.

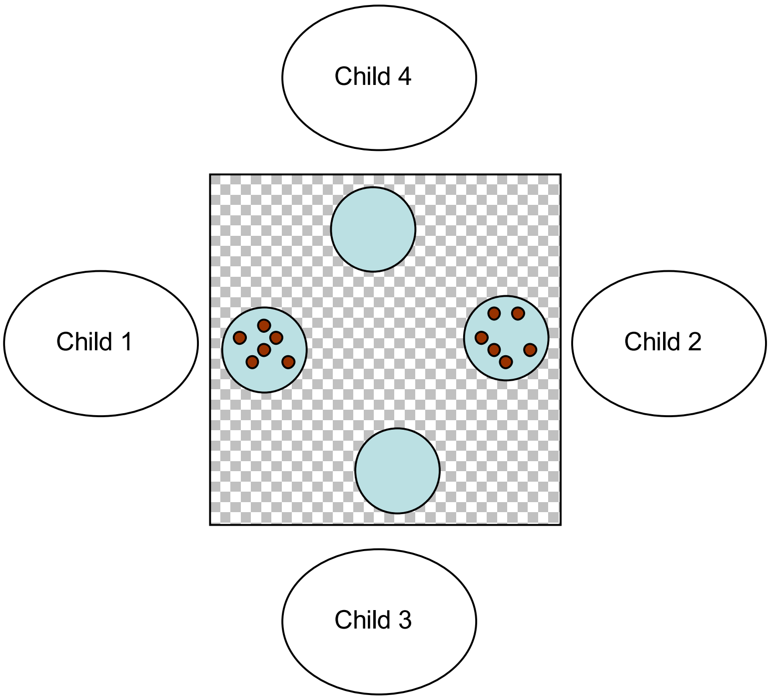


Figure 4. Arrangement of manipulatives.

Strategy 4 - Modifying Educational Games: Games are often used to provide practice for new skills. However, games that provide practice for non-gifted students are likely to be too simple for gifted students who may not need the practice. However, modified games can provide gifted students with learning opportunities. For example, in a simple place value card game, students are dealt three cards in order to represent a three-digit number. They then take turns to select one card at a time from the pile of remaining cards with the goal of making their number larger. For example, if Student A was dealt "345" and the new card was an "8" the optimal position for this card would be in the hundreds to create "845" (See Figure 5).

Student A: (dealt) 3 4 5 → (turned over an '8', elected to replace the 'hundreds card') **8** 4 5

Figure 5. Replacing a card <u>after</u> the new card is known.

This game becomes more challenging, however, if students select the card to be replaced before they see their new card (See Figure 6). As shown by Students B and C's responses, the selection of the card to replace results in substantially different outcomes (See Figure 6). The modified game proved to be a particularly useful thought-revealing activity (Lesh, Hoover, Hole, Kelly, & Post, 2000) because it uncovered gifted students' propensity for calculated risk taking, their erroneous reasoning, and indicators of their metacognitive processes. Thus, this game provided rich learning opportunities for gifted students.

Student B. (dealt) $\boxed{3}\,\boxed{4}\,\boxed{5}$ → (elected to replace the ‘ones’ card) $\boxed{3}\,\boxed{4}\,\boxed{}$ → (turned over an ‘8’) → $\boxed{3}\,\boxed{4}\,\boxed{\mathbf{8}}$

Student C. (dealt) $\boxed{3}\,\boxed{4}\,\boxed{5}$ → (elected to replace the ‘hundreds’ card) $\boxed{}\,\boxed{4}\,\boxed{5}$ → (turned over an ‘8’) → $\boxed{\mathbf{8}}\,\boxed{4}\,\boxed{5}$

Figure 6. Replacing a card <u>before</u> the new card is known.

Concluding Comments

The strategies that have been discussed in this paper are lateral strategies that are designed to expand gifted students’ mathematical knowledge and repertoires of practices through challenging tasks that are connected to the regular curriculum. These lateral strategies have six particular advantages. Firstly, lateral strategies are not add-ons or extensions but take the existing curriculum and problematise, adapt and enrich the experiences for gifted students. Secondly, these approaches lend themselves to both collaborative and independent learning (Diezmann & Watters, 2001). Thirdly, through strong linkages to the regular curriculum, lateral strategies provide underachieving gifted students with opportunities to oscillate between regular tasks and more challenging tasks according to their capability, confidence and motivation. Fourthly, lateral strategies address the mathematics reform agenda, for example through attention to learning through problem solving (Diezmann, Watters, & Thornton, 2003). Thus, strategies to cater for the needs of gifted students can be introduced to preservice and in-service teachers as part of the reform process. Fifthly, because lateral approaches capitalise on primary teachers’ pedagogical knowledge, for example in using games, teachers have opportunities to develop their confidence and competence in working with gifted students. Finally, lateral strategies result in thought-revealing activities that provide teachers with windows into students’ mathematical understandings (Lesh et al., 2000). Hence, they provide opportunities for the identification of gifted students and for the monitoring of the development of their mathematical ability.

The title of this paper, “Challenging Mathematically Gifted Primary Students”, can be interpreted in three ways. Firstly, the title relates to the importance of challenging mathematical gifted primary students. Education plays a fundamental role in the development of aptitude into achievement. Hence, the learning opportunities that a mathematically gifted student has at each year level will presumably either increase or decrease the likelihood that their aptitude in mathematics will be developed and that they will become high achievers in mathematics. Secondly, the title can be interpreted as the challenges for mathematically gifted students. Gifted students face three major challenges, namely the level of the work, isolation, and community attitudes. This paper has addressed the level of work. However, the issue of isolation also needs to be addressed through opportunities for gifted students to work with like-minded peers and to receive adequate teacher attention. As students engage in tasks that are beyond their capabilities, they need the support of others to help them with these tasks, thus, the support of the teacher and gifted peers are important. Gifted students as well as non-gifted students can benefit from scaffolding and social learning opportunities. A further issue that challenges mathematically gifted students is that of community attitudes. Negative community attitudes towards the gifted have been identified in Australia (Collins, 2001). Damarin (2000) proposes that negativity towards the mathematically gifted extends to these individuals being regarded as a deviant population because they find mathematics easy and are passionate about mathematics, which the general populace finds difficult and dislikes. Subsequent to an awareness of the challenges faced by gifted students, is the need to find ways to address these challenges. Finally, the title can be interpreted as the challenges that mathematically gifted primary students provide for teachers. Gifted students can be challenging for primary teachers, however concomitant with the level of challenge is the level of reward. Often gifted individuals retrospectively refer to the interest of particular teachers or the tasks they did in a particular class as key points in their education. How will our mathematically gifted primary students remember us?

A teacher affects eternity. You never can tell where his (or her) influence stops - Henry Adams

References

Baroody, A., & Coslick, R. T. (1998). *Fostering children's mathematical power: An investigative approach in K-8 mathematics instruction.* Mahwah, NJ: Lawrence Erlbaum.

Collins, J. (Chair) (2001). *The education of gifted children.* Canberra: Commonwealth of Australia.

Damarin, S. (2000). The mathematically able as a marked category. *Gender and Education 12*(1), 69-85.

Davis, G. A., & Rimm, S. (2004). Education of the gifted and talented (5th ed.). Boston, MA: Pearson.

Diezmann, C. M., & English, L. D. (2001). Developing young children's multi-digit number sense. *Roeper Review, 24*(1), 11-13.

Diezmann, C. M., & Watters, J. J. (2000). Catering for mathematically gifted elementary students: Learning from challenging tasks. *Gifted Child Today, 23*(4), 14-19.

Diezmann, C. M., & Watters, J. J. (2001). The collaboration of mathematically gifted students on challenging tasks. *Journal for the Education of the Gifted, 25*(1), 7-31.

Diezmann, C. M., & Watters, J. J. (2002a). Summing up the education of mathematically gifted students, In B. Barton, K. C. Irwin, M. Pfannkuch, & M. O. J. Thomas (Eds.), *Proceedings of the 25th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 219-226), Auckland: MERGA.

Diezmann, C. M., & Watters, J. J. (2002b). The importance of challenging tasks for mathematically gifted students. *Gifted and Talented International, 17*(2), 76-84.

Diezmann, C. M., Watters, J. J., & English, L. D. (2002). Teacher behaviours that influence young children's reasoning. In A. D. Cockburn & E. Nardi (Eds.), *Proceedings of the 27th Annual Conference of the International Group for the Psychology of Mathematics Education* (Vol 2, pp. 289-296). Norwich, UK: PME.

Diezmann C, Thornton C., & Watters J. (2003). Addressing the needs of exceptional students through problem solving. In F. Lester & R. Charles (Eds.), *Teaching mathematics through problem solving* (pp. 169-182). Reston, VA: National Council of Teachers of Mathematics.

Gagné, F. (1985). Giftedness and talent: Reexamining a reexamination of the definition. *Gifted Child Quarterly, 29*(3), 103-112.

Gagné, F. (1993). Constructs and models pertaining to exceptional human abilities. In K. A. Heller, F. J. Monks, & A. H. Passow (Eds.), International handbook of research and development of giftedness and talent (pp. 69-87). New York: Pergamon Press.

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education, 28*(5), 524-549.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., Murray, H., Olivier, A., & Wearne, D. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational Researcher, 25*(4), 12-21.

House, P. (Ed.) (1987). *Providing opportunities for the mathematically gifted K-12.* Reston, VA: NCTM.

Howard, J. (2001). *Backing Australia's Ability: Prime Minister's federation speech.* Retrieved 26 July 2004 from http://www.pm.gov.au/news/speeches/2001/speech676.htm

Hutchins, P. (1986). *The doorbell rang.* New York, NY: Greenwillow Books.

Johnson, M. L. (1983). Identifying and teaching mathematically gifted elementary school students. *Arithmetic Teacher 30*(5), 25-26, 55-56.

Krutetskii, J. A. (1976). *The psychology of mathematical abilities in school children.* Chicago, IL: The University of Chicago Press.

Lesh, R. Hoover, M., Hole, B., Kelly, A., & Post, T. (2000). Principles for developing thought revealing activities for students and teachers. In A. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 591-646). Mahwah, NJ: Lawrence Erlbaum.

Lupkowski, A. E., & Assouline, S. G. (1994). *Recognizing and encouraging mathematical talent in students (Up to Year 6).* Australia: Hawker Brownlow.

Marjoram, D. T. E. (1992). Teaching able mathematicians in school. *Gifted Education International 8,* 40-43.

Sheffield, L. J. (1999). *Developing mathematically promising students.* Reston, VA: National Council of Teachers of Mathematics.

Thomas, J. (2000). *Australian science: An investment for the 21st century*. Canberra, ACT: FASTS. Retrieved 26 July 2004 from http://www.FASTS.org/site/policy/occasionalpapers/Lookingfor_future.pdf

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes.* Cambridge, MA: Harvard University Press.